DAY THIRTY THREE

Atoms and Nuclei

Learning & Revision for the Day

- Scattering of α-particles
- Rutherford's Model of the Atom
- Bohr's Model
- Hydrogen Spectrum
- Ionisation Energy and Potential
- Excitation Energy and Potential
- Concept of Nucleus
- Isotopes, Isobars, and Isotones

Atom is the smallest particle of an element which contains all properties of element. **Nuclei** refer to a nucleus of an atom, having a given number of nucleons.

Scattering of α-particles

In 1911, Rutherford successfully explained the scattering of α-particles on the basis of nuclear model of the atom.

Number of α-particles scattered through angle θ is given by

$$N(\theta) \propto \frac{Z^2}{\sin^4 (\theta/2)\, K^2}$$

where, K is the kinetic energy of α-particle and Z is the atomic number of the metal.

At distance of closest approach the entire initial kinetic energy of α-particles is converted into potential energy, so

$$\frac{1}{2} mv^2 = \frac{1}{4\pi\varepsilon_0} \frac{Ze(e)}{r_0} \Rightarrow r_0 = \frac{1}{4\pi\varepsilon_0} \times \frac{2ze^2}{mv^2} = K \times \frac{ze^2}{mv^2}$$

Rutherford's Model of an Atom

On the basis of scattering of α-particles, Rutherford postulated the following model of the atom

- Atom is a sphere of diameter about 10^{-10} m. Whole of its positive charge and most of its mass is concentrated in the central part called the nucleus.
- The diameter of the nucleus is of the order of 10^{-15} m.

- The space around the nucleus is virtually empty with electrons revolving around the nucleus in the same way as the planets revolve around the sun.
- The electrostatic attraction of the nucleus provides centripetal force to the orbiting electrons.
- Total positive charge in the nucleus is equal to the total negative charge of the orbiting electrons.

Rutherford's model suffers from the following drawbacks

(a) stability of the atomic model.

(b) nature of energy spectrum.

Bohr's Model

Bohr's added the following postulates to the Rutherford's model of the atom

- The electrons revolve around the nucleus only in certain permitted orbits, in which the angular momentum of the electron is an integral multiple of $h/2\pi$, where h is the Planck constant $\left(L = mv_n r_n = \dfrac{nh}{2\pi}\right)$.
- The electrons do not radiate energy, while revolving in the permitted orbits, i.e. the permitted orbits are stationary, non-radiating orbits.
- The energy is radiated only when the electron jumps from an outer permitted orbit to some inner permitted orbit. (Absorption of energy makes the electron jump from inner orbit to outer orbit)
- If the energy of the electron in nth and mth orbits be E_n and E_m respectively, then while the electron jumps from nth to mth orbit, the radiation frequency ν is emitted, such that $E_n - E_m = h\nu$.

  This is called the **Bohr's frequency equation**.

NOTE
- Radius of the orbit of electron in a hydrogen atom in its stable state, corresponding to $n = 1$, is called **Bohr's radius**. Value of Bohr's radius is $r_0 = 0.529$ Å ≈ 0.53 Å.
- The time period of an electron in orbital motion in the Bohr's orbit is $T = \dfrac{2\pi r}{v} = \dfrac{2\,\pi \times 0.53}{\dfrac{c}{137}}$ Å $= 1.52 \times 10^{-6}$ s

  and the frequency of revolution is, $f = \dfrac{1}{T} = 6.5757 \times 10^{15}$ cps

Some Characteristics of an Atom

- The **orbital radius** of an electron is

$$r_n = 4\pi\varepsilon_0 \frac{n^2 h^2}{4\,\pi^2 Zme^2} = 0.53\,\frac{n^2}{Z}\ \text{Å}$$

- The **orbital velocity** of an electron is

$$v_n = \frac{1}{4\pi\varepsilon_0}\,\frac{2Z\pi e^2}{nh} = \left(\frac{c}{137}\right)\frac{Z}{n} = 2.2 \times 10^6 \left(\frac{Z}{n}\right)\text{m/s}$$

- **Orbital frequency** is given by $f = \dfrac{1}{T} = \dfrac{v}{2\pi r} = \dfrac{me^4}{4\,\varepsilon_0^2 n^3 h^3}$
- The **total energy** of the orbital electron is

$$E = -\left(\frac{me^4 Z^2}{8\varepsilon_0^2 h^2 n^2}\right)$$

$$= -\left(\frac{me^4}{8\varepsilon_0^2 ch^3}\right) ch\,\frac{Z^2}{n^2}$$

$$= -Rch\,\frac{Z^2}{n^2} = -13.6\,\frac{Z^2}{n^2}\ \text{eV}$$

$$\text{KE} = \frac{me^4 Z^2}{8n^2 h^2\ \varepsilon_0^2}\ ,\quad \text{PE} = -\frac{me^4 Z^2}{4\,n^2 h^2\ \varepsilon_0^2}$$

- The kinetic, potential and total energies of the electron with r as the radius of the orbit are as follows

$$\text{KE} = \frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\,\frac{Ze^2}{r}\right],$$

$$\text{PE} = -\frac{1}{4\pi\varepsilon_0}\,\frac{Ze^2}{r}$$

and $$E = -\frac{1}{2}\left[\frac{1}{4\pi\varepsilon_0}\,\frac{Ze^2}{r}\right]$$

Therefore, they are related to each other as follow

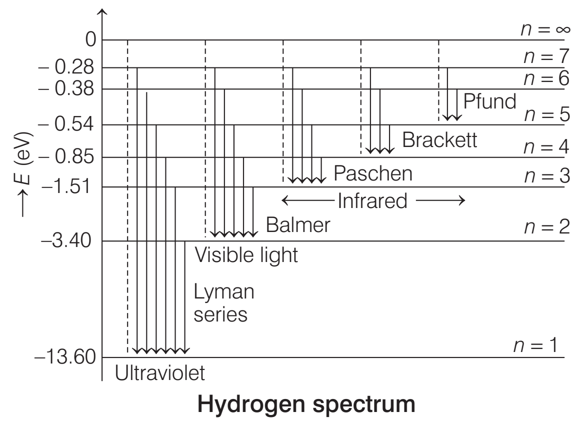
$\text{KE} = -E$ and $\text{PE} = 2E$

- For a hydrogen atom $r_n \propto n^2, v_n \propto \dfrac{1}{n}$ and $|E| \propto \dfrac{1}{n^2}$
- The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is $\Delta L = (n+1)\dfrac{h}{2\,\pi} - \dfrac{nh}{2\,\pi} = \dfrac{h}{2\,\pi}$

Hydrogen Spectrum

Hydrogen spectrum consists of spectral lines classified as five spectral series of hydrogen atom.

Out of these five, Lyman series lies in the ultraviolet region of spectrum, Balmer series lies in the visible region and the remaining three series, lie in the infrared region of spectrum.



Hydrogen spectrum

Total number of emission spectral lines from some excited state n_1 to another energy state $n_2 (< n_1)$ is given by $\frac{(n_1 - n_2)(n_1 - n_2 + 1)}{2}$.

e.g. Total number of lines from $n_1 = n$ to $n_2 = 1$ is $\frac{n(n-1)}{2}$.

The five spectral series of hydrogen atom are given below

1. Lyman Series

Spectral lines of Lyman series correspond to the transition of electron from higher energy levels (orbits) $n_i = 2, 3, 4, \dots$ to ground energy level (1st orbit) $n_f = 1$.

For Lyman series, $\frac{1}{\lambda} = \nu = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right]$,

where $n = 2, 3, 4, \dots$.

It is found that a term $Rch = 13.6$ eV $= 2.17 \times 10^{-18}$ J. The term Rch is known as Rydberg's energy.

2. Balmer Series

Electronic transitions from $n_i = 3, 4, 5, \dots$ to $n_f = 2$, give rise to spectral lines of Balmer series.

Thus, for a Balmer series line, $\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(2)^2} - \frac{1}{n^2}\right]$

where, $n = 3, 4, 5, \dots$.

3. Paschen Series

Lines of this series lie in the infrared region and correspond to electronic transition from $n_i = 4, 5, 6, \dots$ to $n_f = 3$.

Thus, $\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(3)^2} - \frac{1}{n^2}\right]$, where $n = 4, 5, 6, \dots$

4. Brackett Series

It too lies in the infrared region and corresponds to transition from $n_i = 5, 6, 7, \dots$ to $n_f = 4$.

Thus, for Brackett series,

$$\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(4)^2} - \frac{1}{n^2}\right], \text{ where } n = 5, 6, 7, \dots$$

5. Pfund Series

It lies in the far infrared region of spectrum and corresponds to electronic transitions from higher orbits $n_i = 6, 7, 8, \dots$ to orbit having $n_f = 5$. Thus, we have $\frac{1}{\lambda} = \bar{\nu} = R\left[\frac{1}{(5)^2} - \frac{1}{n^2}\right]$, where $n = 6, 7, 8, \dots$

NOTE

- Energy of emitted radiation,

$$\Delta E = E_2 - E_1 = \pm RchZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= 13.6 Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Ionisation Energy and Potential

Ionisation energy of an atom is defined as the energy required to ionise it, i.e. to make the electron jump from its present orbit to infinity. Thus, ionisation energy of hydrogen atom in the ground state $= E_\infty - E_1 = 0 - (-13.6 \text{ eV}) = +13.6$ eV

The potential through which an electron is to be accelerated, so that it acquires energy equal to the ionisation energy is called the ionisation potential.

Therefore, ionisation potential of hydrogen atom in its ground state is 13.6V.

In general, $E_{\text{ion}} = 13.6 \frac{Z^2}{n^2}$ eV or $V_{\text{ion}} = \frac{E_{\text{ion}}}{e}$

Excitation Energy and Potential

Excitation energy is the energy required to excite an electron from a lower energy level to a higher energy level. The potential through which an electron is accelerated, so as to gain requisite ionisation energy is called the ionisation potential.

Thus, first excitation energy of hydrogen atom

$$= E_2 - E_1 = -3.4 - (-13.6) \text{ eV} = +10.2 \text{ eV}$$

Similarly, second excitation energy of hydrogen atom

$$= E_3 - E_1 = -1.51 - (-13.6) = 12.09 \text{ eV}$$

Concept of Nucleus

In every atom, the positive charge and mass is densely concentrated at the centre of the atom forming its **nucleus**. In nucleus, the number of protons is equal to the atomic number of that element and the remaining particles to fulfil the mass number are the neutrons.

Composition of Nucleus

Nucleus consists of protons and neutrons. Electrons cannot exist inside the nucleus. A proton is a positively charged particle having mass (m_p) of 1.007276 u and charge $(+e) = +1.602 \times 10^{-19}$ C.

For a neutral atom,

Number of proton (Z) = Number of electron

This number is called the **atomic number**. A neutron is a neutral particle having mass $m_n = 1.008665$ u. The number of neutrons in the nucleus of an atom is called the **neutron number** N. The sum of the number of protons and neutrons is called the **mass number A.** Thus, $A = N + Z$.

Properties of Nucleus

Nuclear size

(a) Size of the nucleus is of the order of fermi (1 fermi $= 10^{-15}$ m).

(b) The radius of the nucleus is given by $R = R_0 A^{1/3}$, where, $R_0 = 1.3$ fermi and A is the mass number.

Volume

The volume of nucleus is

$$V = \frac{4}{3}\pi (R_0 A^{1/3})^3$$

where, R_0 = radius of the nucleus.

Density

(a) $$\text{Density} = \frac{\text{Mass of nucleus}}{\text{Volume of the nucleus}} = \frac{Am_p}{\frac{4}{3}\pi (R_0 A^{1/3})^3} = \frac{m_p}{\frac{4}{3}\pi R_0^3}$$

where, $m_p = 1.6 \times 10^{-27}$ kg = mass of proton and $R_0 = 1.3$ fermi.

(b) Density of nuclear matter is of the order of 10^{17} kg/m^3.

(c) Density of nuclear matter is independent of the mass number.

Isotopes, Isobars and Isotones

Isotopes

Isotopes of an element are nuclides having same atomic number Z, but different mass number A (or different neutron number N) is called isotopes. ${}_1^1H$, ${}_1^2H$, ${}_1^3H$ and ${}_6^{11}C$, ${}_6^{12}C$, ${}_6^{14}C$, etc., are isotopes.

Isobars

Nuclides having same mass number A, but different atomic number Z are called isobars. In isobars number of protons Z as well as number of neutrons N differ but total nucleon (or mass) number $A = N + Z$ is the same. ${}_1^3H$, ${}_2^3He$ and ${}_6^{14}C$, ${}_7^{14}N$ are isobars.

Isotones

Nuclides with different atomic number Z and different mass number A, but same neutron number are called isotones. ${}_1^3H$, ${}_2^4He$ and ${}_{80}^{198}Hg$, ${}_{79}^{197}Au$ are examples of isotones.

DAY PRACTICE SESSION 1

FOUNDATION QUESTIONS EXERCISE

1 In Rutherford scattering experiment, the number of α-particles scattered at 60° is 5×10^6. The number of α-particles scattered at 120° will be

(a) 15×10^6 (b) $\frac{3}{5} \times 10^6$

(c) $\frac{5}{9} \times 10^6$ (d) None of these

2 In a Rutherford scattering experiment, when a projectile of charge Z_1 and mass M_1 approaches a target nucleus of charge Z_2 and mass M_2, the distance of closest approach is r_0. The energy of the projectile is ➜ CBSE AIPMT 2009

(a) directly proportional to $M_1 \times M_2$

(b) directly proportional to $Z_1 Z_2$

(c) inversely proportional to Z_1

(d) directly proportional to mass M_1

3 In Rutherford experiment, a 5.3 MeV α-particle moves towards the gold nucleus ($Z = 79$). How close does the alpha particle to get the centre of the nucleus, before it comes momentarily to rest and reverses its motion? (Take, $\varepsilon_0 = 8.8 \times 10^{-12}$ F / m)

(a) 3.4×10^{-15} m (b) 8.6×10^{-14} m

(c) 4.5×10^{-13} m (d) 1.6×10^{-14} m

4 The simple Bohr's model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(a) of the electrons not being subject to a central force

(b) of the electrons colliding with each other

(c) of screening effects

(d) the force between the nucleus and an electron will no longer be given by Coulomb's law

5 For the ground state, the electron in the H-atom has an angular momentum = h, according to the simple Bohr's model. Angular momentum is a vector and hence there will be infinitely many orbits with the vector pointing in all possible directions. In actual, this is not true,

(a) because Bohr's model gives incorrect values of angular momentum

(b) because only one of these would have a minimum energy

(c) angular momentum must be in the direction of spin of electron

(d) because electrons go around only in horizontal orbits

6 In the lowest energy level of hydrogen atom, the electron has the angular momentum

(a) $\frac{\pi}{h}$ (b) $\frac{h}{\pi}$ (c) $\frac{h}{2\pi}$ (d) $\frac{2\pi}{h}$

7 Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

(a) 53 pm (b) 27 pm (c) 18 pm (d) 13 pm

8 How many revolutions does an electron complete in one second in the first orbit of hydrogen atom?

(a) 6.57×10^{15} rev/s (b) 100 rev/s
(c) 1000 rev/s (d) 1 rev/s

9 The ratio of kinetic energy to the total energy of an electron in a Bohr orbit of the hydrogen atom, is

(a) $2 : -1$ (b) $1 : -1$ (c) $1 : 1$ (d) $1 : -2$

10 Which of the following transition in hydrogen atoms limit photons of highest frequency?

(a) $n = 1$ to $n = 2$ (b) $n = 2$ to $n = 6$
(c) $n = 6$ to $n = 2$ (d) $n = 2$ to $n = 1$

11 Given that, R is Rydberg's constant. When an electron in an atom of hydrogen jumps from an outer orbit $n = 3$ to an inner orbit $n = 2$, the wavelength of emitted radiations will be equal to

(a) $\frac{R}{6^2}$ (b) $\frac{6^2}{R}$
(c) $\frac{5R}{36}$ (d) $\frac{36}{5R}$

12 An excited hydrogen atom returns to the ground state. The wavelength of emitted photon is λ. The principal quantum number of the excited state will be

(a) $\left(\frac{\lambda R}{\lambda R - 1}\right)^{1/2}$ (b) $\left(\frac{\lambda R - 1}{\lambda R}\right)^{1/2}$

(c) $[\lambda R(\lambda R - 1)]^{1/2}$ (d) $\left[\frac{1}{\lambda R(\lambda R - 1)}\right]^{1/2}$

13 In an inelastic collision, an electron excites a hydrogen atom from its ground state to a M-shell state. A second electron collides instantaneously with the excited hydrogen atom in the M-state and ionises it. At least how much energy the second electron transfers to the atom in the M-state?

(a) + 3.4 eV (b) + 1.51 eV
(c) – 3.4 eV (d) – 1.51 eV

14 A hydrogen like atom emits radiations of frequency 2.7×10^{15} Hz when it makes a transition from $n = 2$ to $n = 1$. The frequency emitted in a transition from $n = 3$ to $n = 1$ will be

(a) 1.6×10^{15} Hz (b) 3.2×10^{15} Hz
(c) 4.8×10^{15} Hz (d) 6.4×10^{15} Hz

15 If an electron in a hydrogen atom jumps from the 3rd orbit to the 2nd orbit, it emits a photon of wavelength λ. When it jumps from the 4th orbit to the 3rd orbit, the corresponding wavelength of the photon will be ➜ NEET 2016

(a) $\frac{16}{25}\lambda$ (b) $\frac{9}{16}\lambda$ (c) $\frac{20}{7}\lambda$ (d) $\frac{20}{13}\lambda$

16 Hydrogen atom in ground state is excited by a monochromatic radiation of $\lambda = 975$ Å. Number of spectral lines in the resulting spectrum emitted will be ➜ CBSE AIPMT 2014

(a) 3 (b) 2 (c) 6 (d) 10

17 The ionisation energy of the electron in the hydrogen atom in its ground state is 13.6 eV. The atoms are excited to higher energy levels to emit radiations of 6 wavelengths. Maximum wavelength of emitted radiation corresponds to the transition between ➜ CBSE AIPMT 2009

(a) $n = 3$ to $n = 2$ states (b) $n = 3$ to $n = 1$ states
(c) $n = 2$ to $n = 1$ states (d) $n = 4$ to $n = 3$ states

18 Monochromatic radiation of wavelength λ is incident on a hydrogen sample. In ground state, hydrogen atom absorbs a fraction of light and subsequently emits radiation of three different wavelengths. The wavelength λ is

(a) 102.73 nm (b) 121.6 nm
(c) 110.3 nm (d) 45.2 nm

19 The limit of Balmer series is 3646 Å. The wavelength of first member of this series will be

(a) 6563 Å (b) 3646 Å
(c) 7200 Å (d) 1000 Å

20 According to Bohr's theory (assuming infinite mass of the nucleus), the frequency of the second line of the Balmer series is

(a) 6.16×10^{14} Hz (b) 6.16×10^{10} Hz
(c) 6.16×10^{13} Hz (d) 6.16×10^{16} Hz

21 In the spectrum of hydrogen, the ratio of the longest wavelength in the Lyman series to the longest wavelength in the Balmer series is ➜ CBSE AIPMT 2015

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{27}{5}$ (d) $\frac{5}{27}$

22 The ratio of wavelengths of the last line of Balmer series and the last line of Lyman series is ➜ NEET 2017

(a) 2 (b) 1
(c) 4 (d) 0.5

23 The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from

(a) $2 \to 1$ (b) $3 \to 2$
(c) $4 \to 2$ (d) $5 \to 3$

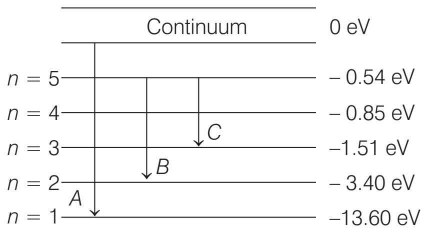
24 The energy of electron in the nth orbit of hydrogen atom is expressed as $E_n = \frac{-13.6}{n^2}$ eV. The shortest and longest wavelength of Lyman series will be

(a) 910 Å , 1213 Å (b) 5463 Å , 7858 Å
(c) 1315 Å , 1530 Å (d) None of these

25 ν_1 is the frequency of the series limit of Lyman series, ν_2 is the frequency of the first line of Lyman series and ν_3 is the frequency of the series limit of the Balmer series. Then,

(a) $\nu_1 - \nu_2 = \nu_3$ (b) $\nu_1 = \nu_2 - \nu_3$
(c) $\frac{1}{\nu_2} = \frac{1}{\nu_1} + \frac{1}{\nu_3}$ (d) $\frac{1}{\nu_1} = \frac{1}{\nu_2} + \frac{1}{\nu_3}$

26. In figure, the energy levels of the hydrogen atom have been shown along with some transitions marked A, B and C. The transitions A, B and C respectively, represents



(a) the first member of the Lyman series, third member of Balmer series and second member of Paschen series
(b) the ionisation potential of H, second member of Balmer series and third member of Paschen series
(c) the series limit of Lyman series, second member of Balmer series and second member of Paschen series
(d) the series limit of Lyman series, third member of Balmer series and second member of Paschen series

27 The ground state energy of hydrogen atom is –13.6 eV. When its electron is in the first excited state, its excitation energy is

(a) 3.4 eV (b) 6.8 eV (c) 10.2 eV (d) zero

28 The energy of a hydrogen atom in the ground state is – 13.6 eV. The energy of a He^+ ion in the first excited state will be ➡ CBSE AIPMT 2010

(a) –13.6 eV (b) –27.2 eV (c) – 54.4 eV (d) – 6.8 eV

29 The ionisation potential of hydrogen atom is 13.6 eV. The energy required to remove an electron from the second orbit of hydrogen will be

(a) 27.4 eV (b) 13.6 eV
(c) 3.4 eV (d) None of these

30 The total energy of the electron orbiting around the nucleus in the ground state of the atom is

(a) less than zero
(b) zero
(c) more than zero
(d) sometimes less and sometimes more than zero

31 The ratio of nuclear radii of the gold isotope ${}_{79}Au^{197}$ and the silver isotope ${}_{47}Ag^{107}$ is

(a) 0.233 (b) 2.33 (c) 1.225 (d) 12.25

DAY PRACTICE SESSION 2

PROGRESSIVE QUESTIONS EXERCISE

1 The total energy of electron in the ground state of hydrogen atom is –13.6 eV. The kinetic energy of an electron in the first excited state is

(a) 3.4 eV (b) 6.8 eV
(c) 13.6 eV (d) 1.7 eV

2 An α-particle after passing through a potential difference of V volt collides with a nucleus. If the atomic number of the nucleus is Z, then the distance of closest approach is

(a) $14.4 \frac{Z}{V}$ Å (b) $14.4 \frac{Z}{V}$ m
(c) $14.4 \frac{V}{Z}$ m (d) $14.4 \frac{V}{Z}$ Å

3 When an electron jumps from a level $n = 4$ to $n = 1$, momentum of the recoiled hydrogen atom will be

(a) 6.8×10^{-27} kg ms^{-1} (b) 12.75×10^{-19} kg ms^{-1}
(c) 136×10^{-19} kg ms^{-1} (d) zero

4 In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^3/2}$ (d) $\frac{1}{n^3}$

5 In the Bohr's model of a hydrogen atom, the centripetal force is furnished by the Coulomb attraction between the proton and the electron. If a_0 is the radius of the ground state orbit, m is the mass and e is the charge on the electron, ε_0 is the vacuum permittivity, the speed of the electron is

(a) zero (b) $\frac{e}{\sqrt{\varepsilon_0 a_0 m}}$
(c) $\frac{e}{\sqrt{4 \pi \varepsilon_0 a_0 m}}$ (d) $\frac{\sqrt{4 \pi \varepsilon_0 a_0 m}}{e}$

6 The binding energy of a H-atom, considering an electron moving around a fixed nuclei (proton), is $B = -\frac{me^4}{8n^2\varepsilon_0^2 h^2}$

(where, m = electron mass)

If one decides to work in a frame of reference, where the electron is at rest, the proton would be moving around it. By similar arguments, the binding energy would be $B = -\frac{Me^4}{8n^2\varepsilon_0^2 h^2}$

(where, M = proton mass)

This last expression is not correct, because

(a) n would not be integral
(b) Bohr quantisation applies only to electron
(c) the frame in which the electron is at rest is not inertial
(d) the motion of the proton would not be in circular orbits, even approximately

7 The recoil speed of hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state is (Take, $R_\infty = 1.097 \times 10^7 \text{m}^{-1}$, $h = 6.63 \times 10^{-34}$ J-s, $M_H = 1.67 \times 10^{-27}$ kg)

(a) 2.2 ms^{-1} (b) 4.18 ms^{-1}
(c) 6.2 ms^{-1} (d) 1 ms^{-1}

8 A hydrogen atom moves with velocity u and makes head on inelastic collision with another stationary H-atom. Both atoms are in ground state before collision. The minimum value of u, if one of the them is to be given a minimum excitation energy is

(a) $2.64 \times 10^4\ \text{ms}^{-1}$ (b) $6.24 \times 10^4\ \text{ms}^{-1}$
(c) $2.64 \times 10^8\ \text{ms}^{-1}$ (d) $6.24 \times 10^8\ \text{ms}^{-1}$

9 Ionisation potential of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1 eV. The spectral lines emitted by hydrogen atoms according to Bohr's theory will be

(a) one (b) two
(c) three (d) four

10 Consider 3rd orbit of He^+ (helium), using non-relativistic approach, the speed of electron in this orbit will be [Take, $K = 9 \times 10^9$ constant, $Z = 2$ and h (Planck constant) $= 6.6 \times 10^{-34}$ J-s] ➜ CBSE AIPMT 2015

(a) 2.92×10^6 m/s (b) 1.46×10^6 m/s
(c) 0.73×10^6 m/s (d) 3.0×10^8 m/s

11 Electron in hydrogen atom first jumps from third excited state to second excited state and then from second excited to the first excited state. The ratio of the wavelengths $\lambda_1 : \lambda_2$ emitted in the two cases is ➜ CBSE AIPMT 2012

(a) 7/5 (b) 27/20 (c) 27/5 (d) 20/7

12 The wavelength of the first line of Lyman series for hydrogen atom is equal to that of the second line of Balmer series for a hydrogen like ion. The atomic number Z of hydrogen like ion is ➜ CBSE AIPMT 2011

(a) 2 (b) 3 (c) 4 (d) 5

13 An electron of a stationary hydrogen atom passes from the fifth energy level to the ground level. The velocity that the atom acquired as a result of photon emission will be ➜ CBSE AIPMT 2012

(a) $\frac{24hR}{25m}$ (b) $\frac{25hR}{24m}$ (c) $\frac{24m}{25hR}$ (d) $\frac{25m}{24hR}$

14 Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^8)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively, for four elements, then approximately which one of the following is correct?

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$
(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

15 The radius of the orbit of an electron in a hydrogen like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck's constant and R is Rydberg's constant. The possible wavelength(λ), when the atom de-excites, is (are)

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$ (c) $\frac{9}{10R}$ (d) $\frac{4}{3R}$

ANSWERS

SESSION 1	1 (c)	2 (b)	3 (c)	4 (a)	5 (a)	6 (c)	7 (c)	8 (a)	9 (b)	10 (a)
	11 (d)	12 (a)	13 (d)	14 (b)	15 (c)	16 (c)	17 (d)	18 (a)	19 (a)	20 (a)
	21 (d)	22 (c)	23 (d)	24 (a)	25 (a)	26 (d)	27 (c)	28 (a)	29 (c)	30 (a)
	31 (c)									
SESSION 2	1 (a)	2 (a)	3 (a)	4 (d)	5 (c)	6 (c)	7 (b)	8 (b)	9 (c)	10 (b)
	11 (c)	12 (a)	13 (a)	14 (c)	15 (a)					

Hints and Explanations

SESSION 1

1 Number of α-particles scattered at angle θ,

$$N \propto \frac{1}{\sin^4 \frac{\theta}{2}}$$

$$\therefore \quad \frac{N_2}{N_1} = \left(\frac{\sin \frac{\theta_1}{2}}{\sin \frac{\theta_2}{2}} \right)^4$$

or $N_2 = 5 \times 10^6 \times \left(\frac{\sin \frac{60^\circ}{2}}{\sin \frac{120^\circ}{2}} \right)^4 = \frac{5}{9} \times 10^6$

2 At the distance of closest approach, the kinetic energy of particle is completely converted to potential energy. Mathematically,

$$\frac{1}{2} M_1 u^2 = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2}{r_0}$$

So, the energy of the particle is directly proportional to $Z_1 Z_2$.

3. The kinetic energy of α-particles is completely converted into potential energy is given by

$$\frac{1}{2} mu^2 = \frac{1}{4\pi\varepsilon_0} \frac{(2e)(Ze)}{r_0}$$

∴ Distance of closest approach,

$$r_0 = \frac{1}{4\pi\varepsilon_0} \frac{2Ze^2}{\frac{1}{2} mu^2}$$

$$= \frac{9 \times 10^9 \times 2 \times 79\,(1.6 \times 10^{-19})^2}{0.5 \times 1.6 \times 10^{-19} \times 10^6}$$

$$= \frac{9 \times 2 \times 79 \times 1.6 \times 1.6 \times 10^9 \times 10^{-38}}{0.5 \times 1.6 \times 10^{-19} \times 10^6}$$

$$= 4.5 \times 10^{-13} \text{ m}$$

4 The simple Bohr's model cannot be directly applied to calculate energy levels of an atom with many electrons. This is because, all the electrons in the atom are not being subjected to one single central force.

5 Bohr's model does not give correct values of angular momentum of revolving electron. It gives only the magnitude of angular momentum, which is a vector. So, the given statement in the question is not true.

6 Angular momentum of electron in nth orbit is given by

$$mvr_n = \frac{nh}{2\pi}$$

In the lowest energy level, $n = 1$.

Then, $\quad mvr_1 = 1\left(\frac{h}{2\pi}\right) = \frac{h}{2\pi}$

7 On the basis of Bohr's model,

$$r = \frac{n^2 h^2}{4\pi^2 m KZe^2} = a_0 \frac{n^2}{Z}$$

For Li^{++} ion, $Z = 3$, $n = 1$ for ground state.

Given, $a_0 = 53$ pm

$$\therefore \quad r = \frac{53 \times 1^2}{3} = 18 \text{ pm}$$

8 By using, $f_n = \dfrac{4\pi K^2 Z^2 e^4 m}{n^3 h^3}$

$$\left[\because f_n = \frac{1}{T_n} = \frac{v_n}{2\pi r_n}\right]$$

$$\therefore f_1 = \frac{12.56 \times 81 \times 10^{18} \times (1.6 \times 10^{-19})^4 \times 9.1 \times 10^{-31}}{(6.62 \times 10^{-34})^3}$$

$$= 6.57 \times 10^{15} \text{ rev/s}$$

9 Kinetic energy of an electron in a Bohr orbit of a hydrogen atom is given as

$$KE_n = \frac{Rhc}{n^2} \quad \text{...(i)}$$

Total energy of an electron in a Bohr orbit of a hydrogen atom is given as

$$TE_n = \frac{-Rhc}{n^2} \quad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{KE_n}{TE_n} = \frac{\left(\frac{Rhc}{n^2}\right)}{-\left(\frac{Rhc}{n^2}\right)}$$

$\Rightarrow \quad KE_n : TE_n = 1 : -1$

10 Energy emitted

$$= E_2 - E_1 = R\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

which is highest for $n_2 = 1$ to $n_1 = 2$.

11 As, $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

Here, $\quad n_2 = 3$ and $n_1 = 2$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left(\frac{1}{(2)^2} - \frac{1}{(3)^2}\right)$$

$$= R\left(\frac{3^2 - 2^2}{3^2 \times 2^2}\right) = R\left(\frac{5}{36}\right)$$

$$\Rightarrow \quad \lambda = \frac{36}{5R}$$

12 $\therefore \quad \dfrac{1}{\lambda} = R\left[\dfrac{1}{1} - \dfrac{1}{n^2}\right]$

or $\quad \dfrac{1}{n^2} = 1 - \dfrac{1}{\lambda R} = \dfrac{\lambda R - 1}{\lambda R}$

or $\quad n = \sqrt{\dfrac{\lambda R}{\lambda R - 1}}$

13 We know that, $E_m = -\dfrac{13.6}{(n)^2}$ eV

$$E_m = -\frac{13.6}{(3)^2} = -1.51$$

Minimum energy required by electron should be – 1.51 eV.

14 $\because \quad \dfrac{1}{\lambda} = \dfrac{\nu}{c} = RZ^2\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$$\Rightarrow \quad \frac{\nu_1}{\nu_2} = \frac{\left[\frac{1}{1} - \frac{1}{4}\right]}{\left[\frac{1}{1} - \frac{1}{9}\right]} \text{ or } \frac{2.7 \times 10^{15}}{\nu_2} = \frac{3 \times 9}{4 \times 8}$$

or $\quad \nu_2 = 3.2 \times 10^{15}$ Hz

15 From Rydberg's formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda'} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$$

$$\therefore \frac{1}{\lambda} \Big/ \frac{1}{\lambda'} = \frac{5R}{36} \div \frac{7R}{144}$$

$$\Rightarrow \quad \frac{\lambda'}{\lambda} = \frac{5R}{36} \times \frac{144}{7R} = \frac{20}{7}$$

$$\Rightarrow \quad \lambda' = \frac{20}{7}\lambda$$

16 Energy incident $= \dfrac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10} \times 1.6 \times 10^{-19}} \text{ eV}$$

$= 12.75$ eV

The hydrogen atom will be excited to $n = 4$

∴ Number of spectral lines $= \dfrac{4(4-1)}{2} = 6$

17 Number of spectral lines,

$$N = \frac{n(n-1)}{2} \quad \Rightarrow \quad \frac{n(n-1)}{2} = 6$$

or $\quad n^2 - n - 12 = 0$

or $\quad (n-4)(n+3) = 0$ or $n = 4$

Now, as the first line of the series has the maximum wavelength, therefore electron jumps from the fourth orbit to the third orbit.

18 As H-atom emits three spectral lines,

$\frac{n(n-1)}{2} = 3$

$\because \quad n = 3$

$\therefore \quad \Delta E = E_4 - E_1 = 13.6 - 1.51$

$= 12.09\,\text{eV}$

$\therefore \quad \lambda = \frac{1242}{12.09} = 102.73\,\text{nm}$

19 $\therefore \quad \frac{1}{\lambda_{\text{limit}}} = R\left[\frac{1}{4} - \frac{1}{\infty}\right]$

$\Rightarrow \quad \frac{1}{\lambda_{\text{first}}} = R\left[\frac{1}{4} - \frac{1}{9}\right]$

$\therefore \quad \lambda_{\text{first}} = 3646 \times \frac{36}{4 \times 5} = 6563\,\text{Å}$

20 Wavelength of Balmer series is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{\nu}{c} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad [\because c = \nu\lambda]$$

$$\nu = cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For second line of Balmer series,

$n_1 = 2; n_2 = 4$

$\nu = 3 \times 10^8 \times 10967800\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$= 6.16 \times 10^{14}\,\text{Hz}$

21 In hydrogen atom, wavelength of characteristic spectrum,

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For maximum wavelength in Lyman series, $n_1 = 1$, $n_2 = 2$

$$\frac{1}{\lambda_1} = RZ^2\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right] \quad \ldots\text{(i)}$$

For maximum wavelength in Balmer series, $n_1 = 2$, $n_2 = 3$

$$\frac{1}{\lambda_2} = RZ^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] \quad \ldots\text{(ii)}$$

Dividing Eq. (ii) by Eq. (i) we get

$$\frac{\lambda_1}{\lambda_2} = \frac{RZ^2\left[\frac{1}{4} - \frac{1}{9}\right]}{RZ^2\left[1 - \frac{1}{4}\right]} = \frac{\frac{5}{36}}{\frac{3}{4}}$$

$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{5}{36} \times \frac{4}{3} = \frac{5}{27}$

22 Wavelength of spectral lines are given by

$$\frac{1}{\lambda} = Z^2R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For last line of Balmer series,

$n_1 = 2$ and $n_2 = \infty$

$\Rightarrow \quad \frac{1}{\lambda_B} = Z^2R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$

$[\because Z = 1]$

Similarly, for last line of Lyman series,

$n_1 = 1$ and $n_2 = \infty$

$\Rightarrow \quad \frac{1}{\lambda_2} = Z^2R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R$

$\therefore \quad \frac{\frac{1}{\lambda_B}}{\frac{1}{\lambda_L}} = \frac{\frac{R}{4}}{R} = \frac{1}{4}$

$\Rightarrow \quad \frac{\lambda_L}{\lambda_B} = \frac{1}{4}$

$\Rightarrow \quad \frac{\lambda_B}{\lambda_\Lambda} = 4$

23 Infrared radiation corresponds to least value of $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$, i.e. from Paschen, Brackett and Pfund series. Thus, the transition corresponds to $5 \to 3$.

24 $\therefore \quad \frac{1}{\lambda_{\max}} = R\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$

$\Rightarrow \quad \lambda_{\max} = \frac{4}{3R} \approx 1213\,\text{Å}$

and $\quad \frac{1}{\lambda_{\min}} = R\left[\frac{1}{(1)^2} - \frac{1}{\infty}\right]$

$\Rightarrow \quad \lambda_{\min} = \frac{1}{R} \approx 910\,\text{Å}$

25 We know that, frequency,

$$\nu = RC\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\nu_1 = RC\left[1 - \frac{1}{\infty}\right] = RC \quad \ldots\text{(i)}$$

$$\nu_2 = RC\left[1 - \frac{1}{4}\right] = \frac{3}{4}RC \quad \ldots\text{(ii)}$$

$$\nu_3 = RC\left[\frac{1}{4} - \frac{1}{\infty}\right] = \frac{RC}{4} \quad \ldots\text{(iii)}$$

On comparing Eqs. (i), (ii) and (iii), we get

$\Rightarrow \quad \nu_1 - \nu_2 = \nu_3$

26 A represents series limit of Lyman series, B represents third member of Balmer series and C represents second member of Paschen series.

27 In the ground state, $n = 1$

$$E_1 = -\frac{13.6}{1^2} = -13.6\text{ eV}$$

For the first excited state (i.e. for $n = 2$),

$$E_2 = \frac{-13.6}{2^2} = -3.4\text{ eV}$$

$\therefore \quad \Delta E = E_2 - E_1 = -3.4 + 13.6$

$= 10.2\text{ eV}$

28 Energy E of an atom with principal quantum number n is given by

$E = \frac{-13.6}{n^2}Z^2$

For first excited state $n = 2$ and for He^+, $Z = 2$

So, $\quad E = \frac{-13.6 \times (2)^2}{(2)^2} = -13.6\,\text{eV}$

29 The potential energy of hydrogen atom,

$$E_n = -\frac{13.6}{n^2}\text{ eV}$$

So, the potential energy in second orbit is

$$E_2 = -\frac{13.6}{(2)^2}\text{ eV} = -3.4\text{ eV}$$

Now, the energy required to remove an electron from second orbit to infinity, is

$U = E_\infty - E_2$

[from work-energy theorem and $E_\infty = 0$]

$\Rightarrow \quad U = 0 - (-3.4)\,\text{eV}$

or $\quad U = 3.4\,\text{eV}$

Hence, the required energy is 3.4 eV.

30 The total energy of the electron orbiting around the nucleus in the ground state of the atom is less than zero.

31 $\because \quad R = R_0A^{1/3}$

$\therefore \quad \frac{R(\text{Au}^{197})}{R(\text{Ag}^{107})} = \left(\frac{197}{107}\right)^{1/3} = (1.841)^{1/3}$

$= \text{antilog}\left[\frac{1}{3}\log(1.841)\right]$

$= \text{antilog}\left[\frac{1}{3} \times 0.2650\right]$

$= \text{antilog}(0.08833) = 1.225$

SESSION 2

1 The energy of hydrogen atom when the electron revolves in nth orbit, is

$E = \frac{-13.6}{n^2}\text{ eV}$

In the ground state, $n = 1$

$$E = \frac{-13.6}{1^2} = -13.6\text{ eV}$$

For $\quad n = 2, \quad E = \frac{-13.6}{2^2} = -3.4\text{ eV}$

So, kinetic energy of electron in the first excited state (i.e. for $n = 2$), is

$$\text{KE} = -E = -(-3.4) = 3.4\text{ eV}$$

2 KE of a particle = 2 eV

$$r = \frac{2(Ze)(e)}{4\pi\varepsilon_0(\text{KE})} = \frac{2Ze^2 \times 9 \times 10^9}{2\text{eV}}$$

$\Rightarrow \quad r = \frac{2 \times Ze \times 9 \times 10^9}{2\text{V}}$

$$\Rightarrow \quad r = \frac{2 \times Z \times 1.6 \times 10^{-19} \times 9 \times 10^{9}}{2\text{ V}}$$

$$= 14.4 \frac{Z}{V} \text{Å}$$

3 $\therefore \quad E = E_4 - E_1 = -\frac{13.6}{4^2} - \left(-\frac{13.6}{1^2}\right)$

$$= -0.85 + 13.6 = 12.75 \text{ eV}$$

$$= 12.75 \times 1.6 = 10^{-14} \text{ J}$$

$$p = \frac{E}{c} = \frac{12.75 \times 1.6 \times 10^{-19}}{3 \times 10^{8}}$$

$$= 6.8 \times 10^{-27} \text{kg ms}^{-1}$$

This must be the momentum of recoiled hydrogen atom
(in opposite direction).

4 $\therefore \Delta E = h\nu$

$$\Rightarrow \nu = \frac{\Delta E}{h} = K\left[\frac{1}{(n-1)^3} - \frac{1}{n^2}\right]$$

$$= \frac{K 2n}{n^2 (n-1)^2} = \frac{2K}{n^3} \propto \frac{1}{n^3}$$

5 From Coulomb's attraction between the positive proton and negative electron

$$= \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} \quad \text{[for neutral atom]}$$

Centripetal force has magnitude,

$$F = \frac{mv^2}{r}$$

So, for the revolving electrons,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$$

$$\Rightarrow v^2 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mr}$$

or $\quad v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$

For ground state of H-atom, $r = a_0$

$$\therefore \quad v = \frac{e}{\sqrt{4\pi\varepsilon_0 m a_0}}$$

6 In a hydrogen atom, electron revolving around a fixed proton nucleus has some centripetal acceleration. Therefore, its frame of reference is non-inertial. In the frame of reference, where the electron is at rest, the given expression cannot be true for binding energy as the frame in which electron is at rest would not be inertial.

7 $\because \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$= (1.097 \times 10^{7})\left(\frac{1}{1^2} - \frac{1}{5^2}\right)$$

$$= 1.053 \times 10^{7} \text{ ms}^{-1}$$

$$\therefore \lambda = 0.95 \times 10^{-7} \text{m}$$

Momentum of photon,

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{0.95 \times 10^{-7}} = 6.98 \times 10^{-27}$$

From conservation of linear momentum,
Momentum of recoiled H-atom = Momentum of photon

$$m_{\text{H}} \times v = \frac{h}{\lambda}$$

$$(1.67 \times 10^{-27}) \times v = 6.98 \times 10^{-27}$$

$$\therefore \quad v = \frac{6.98}{1.67} = 4.18 \text{ ms}^{-1}$$

8 $\quad mu = 2mv$ or $v = \frac{u}{2}$

$$\Delta E = \frac{1}{2}mu^2 - \frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{mu^2}{4}$$

$$\frac{1}{4}mu^2 = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{4}(1.0078)(1.66 \times 10^{-27})u^2$$

$$= 10.2 \times 1.6 \times 10^{-19}$$

$$\Rightarrow \quad u = 6.24 \times 10^{4} \text{ ms}^{-1}$$

9 Energy in excited state

$$= -13.6 + 12.1 = -1.5 \text{ eV}$$

$$\therefore \quad \frac{-13.6}{n^2} = -1.5$$

$$\therefore \quad n = \sqrt{\frac{13.6}{1.5}} = 3$$

Number of spectral lines

$$= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$$

10 Energy of electron in He^+ 3rd orbit

$$= -13.6 \times \frac{Z^2}{n^2} \text{ eV}$$

$$E_3 = -13.6 \times \frac{4}{9} \text{ eV}$$

$$= -13.6 \times \frac{4}{9} \times 1.6 \times 10^{-19} \text{ J}$$

In Bohr's model, $E_3 = -\text{KE}_3$

$$\therefore 9.7 \times 10^{-19} \text{ J} = \frac{1}{2} m_e v^2$$

$$v = \sqrt{\frac{2 \times 9.7 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= 1.46 \times 10^{6} \text{ m/s}$$

11 Here, for wavelength λ_1,

$n_1 = 3$ and $n_2 = 2$

And for λ_2, $n_1 = 2$ and $n_2 = 1$.

We have, $\quad \frac{hc}{\lambda} = -13.6\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$

So, for λ_1,

$$\frac{hc}{\lambda_1} = -13.6\left[\frac{1}{(3)^2} - \frac{1}{(2)^2}\right]$$

$$\frac{hc}{\lambda_1} = 13.6\left[\frac{5}{36}\right] \quad \text{...(i)}$$

Similarly, for λ_2,

$$\frac{hc}{\lambda_2} = -13.6\left[\frac{1}{(2)^2} - \frac{1}{(1)^2}\right]$$

$$\frac{hc}{\lambda_2} = 13.6\left[\frac{3}{4}\right] \quad \text{...(ii)}$$

Hence, from Eqs. (i) and (ii), we get

$$\frac{\lambda_1}{\lambda_2} = \frac{27}{5}$$

12 Lyman series for H-ion,

$$\frac{hc}{\lambda} = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

and for H-like ion,

$$\frac{hc}{\lambda} = Z^2 Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\therefore \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = Z^2\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\left(1 - \frac{1}{4}\right) = Z^2\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\therefore \quad Z = 2$$

13 Here, $\quad E_5 - E_1 = \frac{hc}{\lambda}$

and $\frac{Rhc}{25} - Rhc = \frac{hc}{\lambda} \Rightarrow \frac{24}{25}R = \frac{1}{\lambda}$

But $\quad p = \frac{h}{\lambda}$ and $\nu = \frac{h}{m\lambda} = \frac{24}{25}\frac{Rh}{m}$

14 For hydrogen, we get

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \Rightarrow \frac{1}{\lambda_1} = R(1)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_2} = R(1)^2\left(\frac{3}{4}\right) \Rightarrow \frac{1}{\lambda_3} = R(2)^2\left(\frac{3}{4}\right)$$

$$\frac{1}{\lambda_4} = R(3)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \frac{1}{\lambda_1} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} = \frac{1}{\lambda_2}$$

$$\Rightarrow \lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$$

15 Orbital angular momentum is given by

$$L = 3\left(\frac{h}{2\pi}\right)$$

$$\therefore \quad n = 3, \text{ as } L = n\left(\frac{h}{2\pi}\right)$$

$$r_n \propto \frac{n^2}{Z} \Rightarrow r_3 = 4.5a_0$$

$$\therefore \quad Z = 2$$

$$\frac{1}{\lambda_1} = RZ^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 4R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\Rightarrow \lambda_1 = \frac{9}{5R}$$

$$\frac{1}{\lambda_2} = RZ^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 4R\left(1 - \frac{1}{9}\right)$$

$$\Rightarrow \lambda_2 = \frac{9}{32R}$$

$$\frac{1}{\lambda_3} = RZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 4R\left(1 - \frac{1}{4}\right)$$

$$\Rightarrow \lambda_3 = \frac{1}{3R}$$